Math 1553 Supplement §3.6, 3.7, 3.9, 4.1

1. Suppose V is a 3-dimensional subspace of $\mathbf{R}^5$ containing $\begin{pmatrix}1\\-4\\0\\0\\0\end{pmatrix}$, $\begin{pmatrix}1\\0\\-3\\1\\0\end{pmatrix}$, and $\begin{pmatrix}9\\8\\1\\0\\1\end{pmatrix}$.

$$\text{Must } \left\{\begin{pmatrix}1\\-4\\0\\0\\0\end{pmatrix}, \begin{pmatrix}1\\0\\-3\\1\\0\end{pmatrix}, \begin{pmatrix}9\\8\\1\\0\\1\end{pmatrix}\right\} \text{ be a basis for } V\text{? Justify your answer.}$$

2. Find bases for the column space and the null space of

$$A = \begin{pmatrix} 0 & 1 & -3 & 1 & 0 \\ 1 & -1 & 8 & -7 & 1 \\ -1 & -2 & 1 & 4 & -1 \end{pmatrix}.$$

3. Find a basis for the subspace V of $\mathbf{R}^4$ given by

$$V = \left\{ \begin{pmatrix} x\\y\\z\\w \end{pmatrix} \text{ in } \mathbf{R}^4 \;\middle|\; x + 2y - 3z + w = 0 \right\}.$$

4.
a) True or false: If A is an $m \times n$ matrix and $\mathrm{Nul}(A) = \mathbf{R}^n$, then $\mathrm{Col}(A) = \{0\}$.

b) Give an example of 2×2 matrix whose column space is the same as its null space.

c) True or false: For some m, we can find an $m \times 10$ matrix A whose column span has dimension 4 and whose solution set for $Ax = 0$ has dimension 5.

5. For each matrix A, describe what the transformation $T(x) = Ax$ does to $\mathbf{R}^3$ geometrically.

$$\textbf{a)} \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \textbf{b)} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$