

### Supplemental problems: §2.2, §2.3

1. Put an augmented matrix into reduced row echelon form to solve the system

$$x_1 - 2x_2 - 9x_3 + x_4 = 3$$

$$4x_2 + 8x_3 - 24x_4 = 4.$$

2. a) Row reduce the following matrices to reduced row echelon form.  
b) If these are augmented matrices for a linear system (with the last column being after the = sign), then which are inconsistent? Which have a *unique* solution?

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 7 \\ 6 & 7 & 8 & 9 \end{pmatrix} \quad \begin{pmatrix} 1 & 3 & 5 & 7 \\ 3 & 5 & 7 & 9 \\ 5 & 7 & 9 & 1 \end{pmatrix} \quad \begin{pmatrix} 3 & -4 & 2 & 0 \\ -8 & 12 & -4 & 0 \\ -6 & 8 & -1 & 0 \end{pmatrix}$$

3. We can use linear algebra to find a polynomial that fits given data, in the same way that we found a circle through three specified points in the §2.1 Webwork.

Is there a degree-three polynomial  $P(x)$  whose graph passes through the points  $(-2, 6)$ ,  $(-1, 4)$ ,  $(1, 6)$ , and  $(2, 22)$ ? If so, how many degree-three polynomials have a graph through those four points? We answer this question in steps below.

- a) If  $P(x) = a_0 + a_1x + a_2x^2 + a_3x^3$  is a degree-three polynomial passing through the four points listed above, then  $P(-2) = 6$ ,  $P(-1) = 4$ ,  $P(1) = 6$ , and  $P(2) = 22$ . Write a system of four equations which we would solve to find  $a_0$ ,  $a_1$ ,  $a_2$ , and  $a_3$ .
- b) Write the augmented matrix to represent this system and put it into reduced row-echelon form. Is the system consistent? How many solutions does it have?