

**Math 1553 Worksheet §§5.6-6.3**

1. Courage Soda and Dexter Soda compete for a market of 210 customers who drink soda each day. Today, Courage has 80 customers and Dexter has 130 customers. Each day:

70% of Courage Soda's customers keep drinking Courage Soda, while 30% switch to Dexter Soda.

40% of Dexter Soda's customers keep drinking Dexter Soda, while 60% switch to Courage Soda.

- a) Write a stochastic matrix  $A$  and a vector  $x$  so that  $Ax$  will give the number of customers for Courage Soda and Dexter Soda (in that order) tomorrow. You do not need to compute  $Ax$ .

- b) A quick computation shows that the 1-eigenspace for this positive stochastic matrix  $A$  is spanned by  $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ .

Find the steady-state vector for  $A$ . In the long run, roughly how many daily customers will Courage Soda have?

**2.** Let  $W$  be the set of all vectors in  $\mathbf{R}^3$  of the form  $(x, x - y, y)$  where  $x$  and  $y$  are real numbers.

**a)** Find a basis for  $W^\perp$ .

**b)** Find the matrix  $B$  for orthogonal projection onto  $W$ .