Name:________________________________ Studio Section:____________

Math 1553 Quiz 5, Fall 2019 (10 points, 10 minutes)

Solutions

Show your work on problem 3 or you may receive little or no credit.

1. (1 point each) True or false. If the statement is *always* true, answer TRUE. Otherwise, circle FALSE.

a) If $T : \mathbf{R}^4 \to \mathbf{R}^2$ is a matrix transformation, then T must be onto.

TRUE $\boxed{\text{FALSE}}$

(For example, the zero transformation $T(x_1, x_2, x_3, x_4) = (0, 0)$)

b) Let $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{pmatrix}$. Then the matrix transformation $T(x) = Ax$ is one-to-one.

TRUE $\boxed{\text{FALSE}}$ (A has two columns but only one pivot)

2. (1 point each) In each case, determine whether the transformatino T is linear or not linear. Circle your answers.

a) $T : \mathbf{R}^2 \to \mathbf{R}^3$ given by $T(x_1, x_2) = (x_1,\ x_2 - 1,\ x_1 + x_2)$.

LINEAR $\boxed{\text{NOT LINEAR}}$ $T(0, 0) = (0, -1, 0)$

b) $T : \mathbf{R}^3 \to \mathbf{R}^2$ given by $T(x) = \begin{pmatrix} 0 & 0 & 0 \\ 2 & 2 & 2 \end{pmatrix} x$.

$\boxed{\text{LINEAR}}$ NOT LINEAR (Every matrix transformation is linear)

turn over to the back side for problem 3!

3. (6 points) In this problem, let $T : \mathbf{R}^3 \to \mathbf{R}^3$ be the linear transformation that satisfies

$$T\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}1\\1\\-2\end{pmatrix}, \qquad T\begin{pmatrix}0\\1\\0\end{pmatrix} = \begin{pmatrix}2\\2\\-3\end{pmatrix}, \qquad T\begin{pmatrix}0\\0\\1\end{pmatrix} = \begin{pmatrix}3\\3\\-5\end{pmatrix}.$$

a) Find the standard matrix A for T.

Solution: $A = \begin{pmatrix} T(e_1) & T(e_2) & T(e_3) \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ -2 & -3 & -5 \end{pmatrix}.$

b) Is T one-to-one? If T is one-to-one, justify why. If T is not one-to-one, find vectors x and y (with $x \neq y$) that satisfy $T(x) = T(y)$.

Solution: We row-reduce A:

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ -2 & -3 & -5 \end{pmatrix} \xrightarrow[R_3=R_3+2R_1]{R_2=R_2-R_1} \begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix} \xrightarrow[\text{then } R_1 = R_1 - 2R_2]{R_2 \longleftrightarrow R_3} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}.$$

We see A has three columns but only two pivots, so T is not one-to-one. The parametric form of the solution to $Ax = 0$ is $x_1 = -x_3$, $x_2 = -x_3$, and $x_3 = x_3$ (x_3 is free). So x and y can be any two different vectors in Nul A, for example $x = \begin{pmatrix}0\\0\\0\end{pmatrix}$ and $y = \begin{pmatrix}-1\\-1\\1\end{pmatrix}$, since

$$T(x) = T(y) = \begin{pmatrix}0\\0\\0\end{pmatrix}.$$

There are many other possible answers for x and y, and in this problem row-reduction was actually not necessary. For example, we can see

$$T(e_1 + e_2) = T(e_3)$$

just by looking at the formula for T. This shows T is not one-to-one since

$$T(x) = T(y) \text{ for } \quad x = e_1 + e_2 = \begin{pmatrix}1\\1\\0\end{pmatrix} \quad \text{and} \quad y = e_3 = \begin{pmatrix}0\\0\\1\end{pmatrix}.$$