Name:______________________________________ Studio Section:______________

Math 1553 Quiz 3, Fall 2019 (10 points, 10 minutes)

Solutions

Show your work on problem 4 or you may receive little or no credit.

1. (2 points) Suppose $v_1, v_2, \ldots, v_k$ be vectors in $\mathbf{R}^n$. Give a mathematically precise definition of Span$\{v_1, v_2, \ldots, v_k\}$.

Solution: In set-builder notation,

$$\text{Span}\{v_1, v_2, \ldots, v_k\} = \left\{ x_1 v_1 + \cdots + x_k v_k \;\middle|\; x_1, \ldots, x_k \text{ real} \right\}.$$

(the dividing bar could be a colon if desired, and "real" could be "scalar" here; that is just a matter of notation)

Alternatively, the student could state that Span$\{v_1, v_2, \ldots, v_k\}$ is the set of all linear combinations of $v_1, v_2, \ldots, v_k$.

2. (2 points) Consider the following linear system of equations in x_1, x_2, x_3:

$$\begin{aligned} x_1 - 2x_2 + x_3 &= 1 \\ x_2 - x_3 &= -4 \\ x_1 + x_2 &= 5. \end{aligned}$$

Write this system as a vector equation.

Solution:

$$x_1 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + x_2 \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} + x_3 \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ -4 \\ 5 \end{pmatrix}.$$

3. (1 point) True or false. If the statement is always true, circle TRUE. Otherwise, circle FALSE.

a) If v_1 and v_2 are vectors in $\mathbf{R}^3$, then $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ must be in Span$\{v_1, v_2\}$.

TRUE. No matter what v_1 and v_2 are, $0v_1 + 0v_2 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$.

b) $\text{Span}\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right\} = \mathbf{R}^2.$

FALSE. The two vectors are in $\mathbf{R}^3$, so it would make no sense whatsoever to say their span is equal to $\mathbf{R}^2$.

4. (4 points) Write $\begin{pmatrix} 2 \\ -8 \end{pmatrix}$ as a linear combination of $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$.

Solution: We solve $x_1 \begin{pmatrix} 1 \\ 2 \end{pmatrix} + x_2 \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \\ -8 \end{pmatrix}$:

$$\left(\begin{array}{cc|c} 1 & 1 & 2 \\ 2 & -1 & -8 \end{array}\right) \xrightarrow{R_2 = R_2 - 2R_1} \left(\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & -3 & -12 \end{array}\right) \xrightarrow[\text{then } R_1 = R_1 - R_2]{R_2 = -R_2/3} \left(\begin{array}{cc|c} 1 & 0 & -2 \\ 0 & 1 & 4 \end{array}\right).$$

Thus $x_1 = -2$ and $x_2 = 4$. Our answer is

$$-2\begin{pmatrix} 1 \\ 2 \end{pmatrix} + 4\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \\ -8 \end{pmatrix}.$$