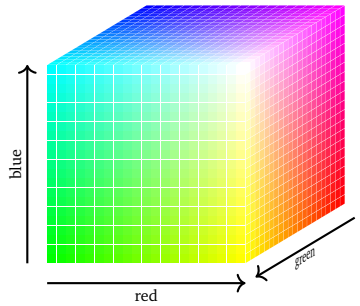
Math 1553 Worksheet §3.5-3.7, 3.9, 4.1

1. Every color on my computer monitor is a vector in $\mathbf{R}^3$ with coordinates between 0 and 255, inclusive. The coordinates correspond to the amount of red, green, and blue in the color.



Given colors $v_1, v_2, \ldots, v_p$, we can form a "weighted average" of these colors by making a linear combination

$$v = c_1 v_1 + c_2 v_2 + \cdots + c_p v_p$$

with $c_1 + c_2 + \cdots + c_p = 1$. Example:

$\frac{1}{2}$  $+$ $\frac{1}{2}$  $=$ 

Consider the colors on the right. Are these colors linearly independent? What does this tell you about the colors?

$\begin{pmatrix} 240 \\ 140 \\ 0 \end{pmatrix}$  $\begin{pmatrix} 0 \\ 120 \\ 100 \end{pmatrix}$  $\begin{pmatrix} 60 \\ 125 \\ 75 \end{pmatrix}$ 

After doing this problem, check out the interactive demo, where you can adjust sliders to find a prescribed color.

2. Circle **TRUE** if the statement is always true, and circle **FALSE** otherwise.

a) If A is a 3×100 matrix of rank 2, then $\dim(\text{Nul}A) = 97$.

TRUE **FALSE**

b) If A is an $m \times n$ matrix and $Ax = 0$ has only the trivial solution, then the columns of A form a basis for $\mathbf{R}^m$.

TRUE **FALSE**

c) The set $V = \left\{ \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \text{ in } \mathbf{R}^4 \;\middle|\; x - 4z = 0 \right\}$ is a subspace of $\mathbf{R}^4$.

TRUE **FALSE**

3. Let $A = \begin{pmatrix} 1 & -5 & -2 & -4 \\ 2 & 3 & 9 & 5 \\ 1 & 1 & 4 & 2 \end{pmatrix}$, and let T be the matrix transformation associated to A, so $T(x) = Ax$.

a) What is the domain of T? What is the codomain of T? Give an example of a vector in the range of T.

b) The RREF of A is $\begin{pmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$. Is there a vector in the codomain of T which is not in the range of T? Justify your answer.