Name:________________________ Recitation Section:__________

Math 1553 Quiz 3, Fall 2018: Sections 3.1 and 3.2 (10 points, 10 minutes)

Solutions

Show your work unless instructed otherwise! A correct answer without appropriate work will receive little or no credit.

1. (2 points) Complete the following mathematical definition of linear combination (Be precise! You cannot use "span" in the definition of linear combination).
Let $w, v_1, v_2 \dots, v_p$ be vectors in $\mathbf{R}^n$. We say w is a *linear combination* of $v_1, v_2 \dots, v_p$ if...

$w = c_1 v_1 + c_2 v_2 + \cdots + c_p v_p$ for some scalars $c_1, \dots, c_p$.
(also fine: "real numbers" rather than "scalars")

2. (3 points) True or false. Circle TRUE if the statement is always true. Otherwise, circle FALSE. You do not need to justify your answer.

a) $\text{Span}\left\{\begin{pmatrix}5\\2\end{pmatrix}\right\}$ contains the zero vector $\begin{pmatrix}0\\0\end{pmatrix}$. $\boxed{\text{TRUE}}$ FALSE

Taken almost directly from the 3.1-3.2 supplement.

b) $\text{Span}\left\{\begin{pmatrix}1\\2\\1\end{pmatrix}, \begin{pmatrix}-2\\-4\\-2\end{pmatrix}\right\}$ is a plane. TRUE $\boxed{\text{FALSE}}$

The second vector is a scalar multiple of the first, so they lie on the same line through the origin. Their span is just $\text{Span}\left\{\begin{pmatrix}1\\2\\1\end{pmatrix}\right\}$, which is a line.

c) Determining whether a vector equation $x_1 v_1 + x_2 v_2 = b$ has a solution is the the same as determining whether v_1 is in $\text{Span}\{v_2, b\}$.

TRUE $\boxed{\text{FALSE}}$

From class and the T/F on Webwork, we know the vector equation has a solution if and only if b is in $\text{Span}\{v_1, v_2\}$, so something should look very amiss right away. For a particular example why the answer is FALSE, note that $0\begin{pmatrix}1\\0\\0\end{pmatrix} + 1\begin{pmatrix}0\\1\\0\end{pmatrix} = \begin{pmatrix}0\\1\\0\end{pmatrix}$ even though $\begin{pmatrix}1\\0\\0\end{pmatrix}$ is not in the span of $\begin{pmatrix}0\\1\\0\end{pmatrix}$.

3. (5 pt) Find all values of h so that $\begin{pmatrix} -1 \\ -7 \\ h \end{pmatrix}$ is a linear combination of $\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix}$.

Show your work!

Solution.

We need to find all values of h so that the augmented system below is consistent.

$$\left(\begin{array}{cc|c} 1 & -2 & -1 \\ -2 & 1 & -7 \\ 1 & 3 & h \end{array}\right) \xrightarrow[R_3=R_3-R_1]{R_2=R_2+2R_1} \left(\begin{array}{cc|c} 1 & -2 & -1 \\ 0 & -3 & -9 \\ 0 & 5 & h+1 \end{array}\right) \xrightarrow{R_2=-R_2/3} \left(\begin{array}{cc|c} 1 & -2 & -1 \\ 0 & 1 & 3 \\ 0 & 5 & h+1 \end{array}\right)$$

$$\xrightarrow{R_3=R_3-5R_2} \left(\begin{array}{cc|c} 1 & -2 & -1 \\ 0 & 1 & 3 \\ 0 & 0 & h-14 \end{array}\right).$$

The system is consistent if and only if the right hand column is not a pivot column, which means we need $h-14=0$, thus $\boxed{h=14}$.